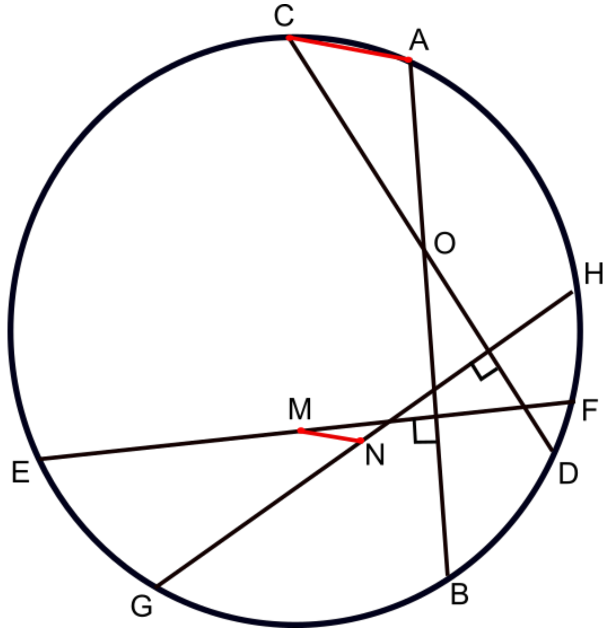
Cash Award Question of May-2025



In the picture, AB & CD are two random chords of the circle intersecting each other at O. EF & GH are the perpendicular bisectors of the segments OB & OD respectively and M & N are the midpoints of EF & GH respectively.

Prove: MN parallel CA.

Question framed by
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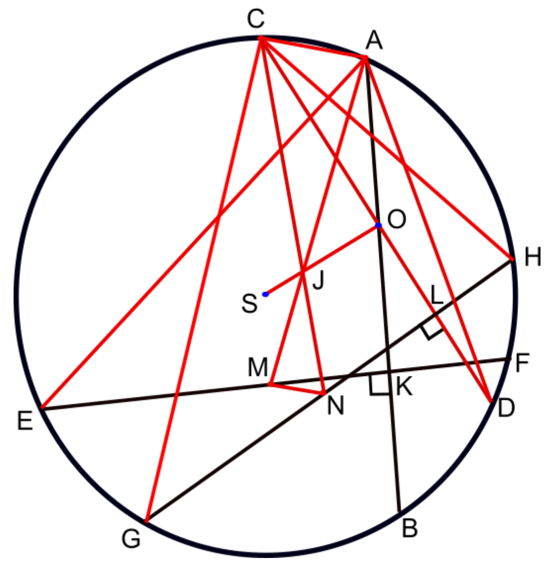
Author's Solution May - 2025

Solution

Given :

AB & CD are two random chords of the circle intersecting each other at O. EF & GH are the perpendicular bisectors of the segments OB & OD respectively and M & N are the midpoints of EF & GH respectively.



To prove : **MN|| CA**

Construction :

Mark the circumcentre as S. Let AB &EF cut at K. Let CD & GH cut at L. Join AF, AE, CG, CH, AM, CN & OS. Let J be a point on OS such that $\frac{OJ}{JS} = \frac{2}{1}$.

Proof:

In $\Delta AEF, AK$ is altitude.

OK = KB (given)

∴ As per the Orthocenter Theorem [(page 51 of the book "Advanced Theorems on Geometry" available in this site], O is the orthocentre ofΔAEF.

Similarly, OL=LD.

⟹ O is also the orthocentre of ΔCGH.

For both the triangles, the orthocentre (O) and the circumcentre(S) are same. Hence, OS is the Euler line for both the triangles and J divides OS in the ratio of 2:1.

$\Longrightarrow$ J is the Centroid for both the triangles.

$\Longrightarrow AM \& \ CN$ intersect each other at J, their common Centroid.

$\Longrightarrow \dfrac{AJ}{JM} = \dfrac{CJ}{JN} = 2.$

$\Longrightarrow$ AC ∥ MN --------------------- Proved.

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Corollary : AC= 2 MN

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